

Application of Deep Neural Network for Stock Value Prediction and Analysis from Text Data

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ABSTRACT:

The recent advance of deep learning has enabled trading algorithms to predict stock price movements more accurately. Unfortunately, there is a significant gap in the real-world deployment of this breakthrough. For example, professional traders in their long-term careers have accumulated numerous trading rules, the myth of which they can understand quite well. On the other hand, deep learning models have been hardly interpretable. This paper presents Deep Clue, a system built to bridge text-based deep learning models and end users through visually interpreting the key factors learned in the stock price prediction model. We make three contributions in DeepClue. First, by designing the deep neural network architecture for interpretation and applying an algorithm to extract relevant predictive factors, we provide a useful case on what can be interpreted out of the prediction model for end users. Second, by exploring hierarchies over the extracted factors

and displaying these factors in an interactive, hierarchical visualization interface, we shed light on how to effectively communicate the interpreted model to end users. Specially, the interpretation separates the predictable from the unpredictable for stock prediction through the use of intercept model parameters and a risk visualization design. Third, we evaluate the integrated visualization system through two case studies in predicting the stock price with online financial news and company-related tweets from social media. Quantitative experiments comparing the proposed neural network architecture with state-of-the-art models and the human baseline are conducted and reported. Feedbacks from an informal user study with domain experts are summarized and discussed in details. All the study results demonstrate the effectiveness of DeepClue in helping to complete stock market investment and analysis tasks.

1. INTRODUCTION

DEEP learning techniques [1] are reshaping the landscape of predictive analysis in the big data research area and have made major breakthroughs in image and speech recognition [2], question answering [3], machine translation [4] and many other application domains. In this paper, we focus on the financial analytics domain. It has been shown that texts such as financial news and tweets on stock markets are useful in predicting stock price movements. For example, financial news such as “Amazon profit beats forecasts” was accompanied with a surge of Amazon’s stock price, while “Oil price hits a record high” triggered worries on the auto industry and weakened their performance in the stock market. Previous work has

demonstrated an over 60% accuracy in predicting the daily stock price movement using deep neural networks over a large collection of financial news. Nevertheless, end users can hardly benefit from these successful deep learning models in their primitive form. We consider two classes of users in this work: stock traders from public/private funds (or independent investors) who manage the stock trading operations; and stock market analysts, who provide the stock prediction models for traders. First, the everyday job of traders is to make trading decisions, i.e., to buy/sell which stock at which particular time. Such a decision is typically based on multiple sources of information known as trading signals, coming out of a large number of trading rules accumulated in the long term. To cope with the trader’s job, there should be a method to help traders detect signals from the prediction model, so that traders can combine these signals with their traditional source of information to finalize the decision. The automatic stock trading based only on the prediction model can be an option,

but it will require a much higher accuracy than that of the latest model. In some cases, a close to 60% accuracy can even lead to losses (Section 6.2). On the other hand, analysts’ work is to fine-tune the stock price prediction model for particular stocks and market trends, in order to optimize the prediction accuracy. This will require analysts to have a deep understanding of failure cases of the prediction model. To this end, both classes of end users will benefit from deep learning technology only if they can interpret the prediction model on where, when and why it works or does not work. This knowledge can then be assembled with the domain expertise to improve the investment in the stock market. Unfortunately, on interpretability deep learning models suffer from a well-known drawback in contrast to traditional machine learning methods such as linear regression and support vector machines (SVM). In some areas such as image recognition, the mechanism of deep learning has been partially known, e.g., working as level-of-detail feature selectors, from the basic visual feature up to motifs and finally to objects [8] [9]. For most other domains, there is still little clue on how deep learning models

work. In our scenario, the use of text input introduces an additional work embedding stage to map text collections onto the feature space, which makes it more difficult to interpret the prediction model. In this paper, we target the research problem of how to interpret text-based deep stock prediction model for end users, so that they can make up their stock trading decisions as well as improve the prediction model based on the interpretation. In particular, we investigate research questions including what kind of information can be efficiently extracted from the prediction

model as interpretations, and how to communicate such information in an effective way to end users. Throughout this work, we depend on an interactive visualization interface to bridge the prediction model and end users, which turns out a natural and straightforward choice. Yet, designing and prototyping such a visualization system can be quite

challenging. First, traditional patterns discovered from data can be presented by visually distinct channels in the same data view, while in this case, the information extracted from the model lies in a higher order than the data pattern. Multiple coordinated views should be

designed elaborately to illustrate the relationship among data, model, and interpretation. Second, the deep learning model is designed in a bottom-up structure to take advantage of the machine's capability in processing huge amount of data, while the visual information-seeking mantra is "overview first, details on demand" [10]. Third, it is commonly accepted that the stock market is information efficient [11], but not all stock price movements are predictable or reflected in text

information. Ingenuity is required to separate predictable and unpredictable price changes. In the literature, there is a recent surge on the topic

of visualizing deep neural networks (DNN) for model interpretation. A large portion of these methods focused on the display of neural network architecture to

help users understand the functionality of individual neurons and features [8] [12], interpret the mechanism of both small-scale neural networks [13] and large-scale multi-layer DNNs [14]. Another thread of research proposed to visualize the model output (e.g. the image class model [15]) or their correspondence to the input data through algorithms similar to back propagation [9]. While our study aligns with these successful methods on DNN model interpretation, the goal is fundamentally different. Instead of visually illustrating DNN structures, we target at extracting useful information from the prediction model, and incorporating this interpretation with domain expertise to improve the performance of stock trading and modeling. In addition, existing literature mostly studied model interpretation for image recognition and object detection tasks, while to our knowledge, we are the first to visually interpret the hidden linkage between public text collections and stock prices through deep learning models. In summary, we make the following contributions.

1. Based on a customized DNN architecture for stock price prediction (Section 3), we apply a model interpretational algorithm, i.e., the pixel-based layer-wise relevance propagation [16], to extract the textual factors relevant to the daily prediction result (Section 4.1). Notably, the extracted factors (i.e., keywords, bigrams, titles) are analyzed to form a factor hierarchy for effective visual interpretation by end users (Section 4.2).

2. An integrated visualization system called DeepClue is designed and applied to the stock price prediction scenario, which visually correlates algorithm-extracted textual factors with stock price movements and the risks associated with the text-based prediction. Flexibilities are granted to end users in model configuration, factor analysis, and detailed reasoning. (Section 5)

3. We evaluate the proposed system through real-life cases in analyzing the text-based deep stock prediction model built from financial

news (Section 6.1) and social media collections (Section 6.2) on US stock markets. Quantitative experiments are conducted to compare the proposed neural network architecture with state-of-the-art models and the human baseline (Section 6.3). Informal user studies are then carried out with private-fund stock traders and deep learning model builders, which demonstrate the value of DeepClue in optimizing stock trading operations and improving the prediction model of stock price movement (Section 6.4).

2 RELATED WORKS

DNN Interpretation and Visualization

Early research related to the DNN interpretation can be found in Erhan et al. [8], who introduced the concept of understanding a particular unit of DNN by visualizing input that maximizes the unit's response. This activation maximization method was compared with other alternatives including sampling the unit and linear combination of previous filters. Experiment results on image data sets showed that the activation maximization methods produced more interesting interpretation results. Zeiler et al. [9] proposed to map feature activations in neural nets back to inputs by deconvolution layers. Simonyan et al. [15] developed a class model visualization that generates a representative image for each class of interest, and a class saliency map for a single input image based on gradients with respect to the input pixel. Bach et al. [16] [17] introduced a class of algorithms named layer-wise relevance propagation (LRP), which decompose a neural net prediction layer by layer into scores for each neural unit, and applied it to state of the art deep networks in image classification. These scores, when computed for the inputs, explain the amount of contribution of a pixel or region to the prediction value for a given class. Dudovskiy et al. [18] trained neural networks

to reconstruct inputs from feature representations. Zintgraf et al. [19] developed an elaborate conditional sampling algorithm to analyze how deep neural networks respond to perturbed inputs. Yosinski et al. [12] introduced tools to visualize the activations on neural network layers, and the features extracted at each layer through regularization. Liu et al. proposed CNNVis [14], a visual analytics approach that employs layer and neuron clustering. CNNVis introduced several novel visualization algorithms such as hierarchical rectangle packing and matrix reordering to display features on clustered neurons. Visualizations have made their way into deep learning toolboxes. Besides the well-known deep dream [20], TensorFlow Playground by Google [13] provided an online visualization tool for non-experts to understand deep learning architecture and their training process through a direct manipulation design. Overall, previous

literature on deep learning model visualization concentrates on the scenario of image classification with CNNs. DeepClue, in contrast to these systems, is dedicated to stock market investors for better understanding the association between text streams and stock price time series. Moreover, rather than opening the Blackbox structure of neural networks and interpreting the functionality of each individual unit, our method focuses on extracting input level interpretable information from the DNN model and visually incorporating such information with domain expertise to improve the performance of stock trading and modeling.

Text-based Stock Prediction and Visualization

It has been pointed out by Kearney and Liu [21] that the complex and time-varying relationship between textual information and

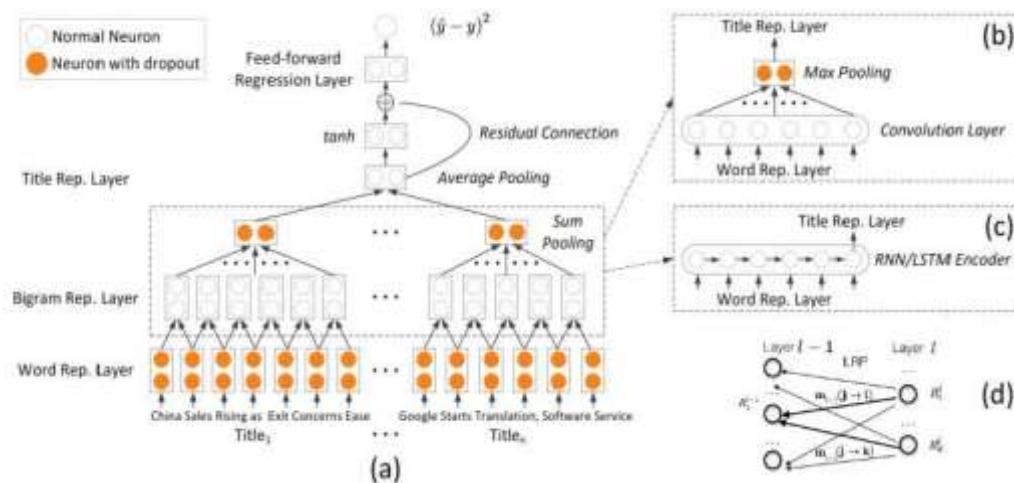


Fig. 1: Neural network architecture in this work: (a) the overall hierarchical structure featuring multiple representation layers from single word to bigram and news/tweet title; (b) an alternative convolution layer that can replace the bigram representation and the sum pooling on each title; (c) another design by a recurrent neural network layer with LSTM cells to represent each title as time series; (d) the mechanism of LRP algorithm that propagates relevance scores back to the input features over the neural network.

stock price poses an important area of study for financial analysis. The textual data for stock price prediction comes primarily from three data sources: public corporate filings, news articles, and the emerging social media content. This work focuses on the latter two sources. On

using news articles for the prediction, Engelberg [22] and Tetlock et al. [23] employed firm-specific news from multiple sources to predict firms' fundamentals, which inherently influence their stock prices. In addition, social media content,

especially textual sentiments, has shown implicit effects on the stock market.

The work by Chen et al. revealed that the views expressed on a popular social media site for investors have strong associations with the related firm's stock returns, thus helps to predict their stock price changes [24]. On stock market visualization, most state-of-the-art literature focused on the display of stock price time series. Similarity and cluster analysis have been employed to group the stock price time series into trajectories to optimize the visualization. For example, Ziegler et al. visually analyzed the distribution of time series trajectories among different market sectors [25]. Keim et al. presented the Growth Matrix visualization [26] for the simultaneous display of growth rates of all possible subintervals in a time series. Beyond the time series visualization, many other designs incorporate the related news and events to the display of stock price time series. Contexture [27] produced annotated stock price visualizations given news articles as the information source. Sorenson and Brath proposed a system to visualize a large collection of stock-related events in a single view [28]. The event display can be visually correlated with the stock price time series for reasoning. Compared to the DeepClue visualization, most above works feature a direct visualization of raw stock price time series and the temporally correlated news/events. There has been little research on visualizing the predictive linkage between the stock price and the textual information, which is extracted from a state-of-the-art deep learning model.

3 TEXT-BASED STOCK PRICE PREDICTION

Data Collection

We consider S&P 500 stocks in the US stock market from 2006 to 2015. Their historical prices are acquired from Yahoo Finance. We crawled financial news from Reuters and Bloomberg, obtaining in total 341,310 news

articles. For each news, we extracted the title, textual content, and timestamp from their raw HTML file. To map each news to the corresponding stocks, we maintained a list of keywords for each firm (e.g., Apple: AAPL, AAPL.O, APPLE, AAPL.N, Apple Inc, etc.). The stock-related tweets were collected through Twitter API in a period from April 2015 to November 2015, by matching the firm's cashtags in the tweet content. Cashtag [29] is a new way of sharing financial information on social media developed by Twitter and other providers. The firm's stock ticker symbols are prefixed with a dollar sign to compose the cashtag, e.g., Apple=\$AAPL, Google=\$GOOG. In total, we obtained 6,869,771 stock-related tweets. For each tweet, we recorded the create time, textual content, source, user, location and related firms.

Deep Neural Network Architecture

We take news data as an example to introduce the architecture of the neural network model adopted in this work. The model is built for each particular S&P 500 firm. The goal of the model is to predict a stock price \hat{y} that is close to the real stock price y of the firm. The raw input of each model is the set of financial news titles collected on the target firm. Intuitively, news content can be useful for further enhancing the prediction accuracy. However, preliminary experiments using both news title and content as inputs (Supplemental Material—Table I) show that our model does not benefit from the additional content information, compared with only using the news title (Figure 11(a)). This is consistent with the observations of Ref. [5], who extract event information from both news title and content, showing that it does not substantially improve a model with only news title as the information source. Therefore, we leave it to future work to further exploit the usefulness of news content information. Figure 1(a) shows our proposed deep regression model organized in a

hierarchical neural network structure. The network consists of four layers: a word representation layer, a bigram representation layer, a title representation layer, and a feed-forward regression layer. The word representation layer accepts all the news titles as input and turns each word in the title into a real-valued word embedding vector [30]. The bigram representation layer constructs representation vectors for word bigrams based on the representation vector of individual words. The title representation layer summarizes representations of word bigrams and encodes each title into a title vector. The feed-forward regression layer receives the output of the title encoder and maps the output to a real-valued prediction through a feed-forward neural network with residual connections [31]. In addition to the prediction, the proposed model is also optimized for the interpretation purpose by three key designs. First, we explicitly extract representation vectors (i.e., features) from the input news titles in a hierarchical, interpretable way (word! bigram ! title), which provides the opportunity to efficiently visualize a large amount of contributing factors. Second, we make use of a combination of techniques to prevent overfitting, e.g. the dropout mechanism. Third, as the hierarchical method lengthens the backward propagation path, we introduce residual connections to ease the burden of training a deep neural network. Note that the proposed deep stock prediction model can be upgraded by introducing state-of-the-art deep neural network structure, such as Convolutional Neural Network (CNN) and Recurrent Neural Network (RNN). In Section 6.3, we will describe two such alternative designs by replacing the bigram-based title representation method with convolution layers (Figure 1(b)) and RNN with LSTM cells (Figure 1(c)). The prediction performance of these alternative designs is also studied in Section 6.3.

4 MODEL INTERPRETATION

4.1 Relevant Keyword Extraction

We introduce a method to identify the importance of textual factors to the stock price change by analyzing the neural network model. The goal is to compute a relevance score with respect to the prediction result of each trading day, denoted as $f(\cdot)$, for each word, bigram, and news title. Take the word relevance score as an example, $f(v)$ defines how much contribution a word with the vector representation v has made to the stock price prediction. A positive (negative) score indicates that this word is evidence for the rise (fall) of the stock price.

$$f = \sum_i f_i^l \quad (5)$$

$$\Delta f_{i \rightarrow j}^{l-1} = \frac{m_{l-1}(j \rightarrow i)}{\sum_k m_{l-1}(k \rightarrow i)} f_i^l \quad (6)$$

$$f_j^{l-1} = \sum_i \Delta f_{i \rightarrow j}^{l-1} \quad (7)$$

$$f(s_k) = f(h) \cdot \frac{w_k s_k}{(\mathbf{w} \times \mathbf{s} + b) + \varepsilon \cdot \text{sgn}(\mathbf{w} \times \mathbf{s} + b)} \quad (8)$$

$$f(b) = f(h) \cdot \frac{b}{(\mathbf{w} \times \mathbf{s} + b) + \varepsilon \cdot \text{sgn}(\mathbf{w} \times \mathbf{s} + b)} \quad (9)$$

Finally, the overall relevance score of each word and bigram is obtained by summing up their relevance scores propagated from all the titles at all vector dimensions. Note that each relevance score is computed once every day according to the daily prediction result.

4.2 Factor Analysis

By the LRP algorithm, a list of words having nonzero relevance score can be obtained on each day, which is defined as key-words here. These keywords, together with relevant bigrams and news/tweet titles, compose the potential influencing factors to the change of stock price. A straightforward method to visualize these factors is to juxtapose the top factors and their relevance score time series in a list view. This view can be aligned with the time series of the actual/predicted stock price changes for multi-factor analysis. Due to the large number of relevant factors (e.g.,

1801 keywords for Apple/APPL), there is an obvious constraint that the list view can quickly grow beyond the limit of the screen space. These keywords can be shortlisted after sorting by the overall relevance (i.e., ℓ_1 norm of the relevance vector), but an overview of all relevant factors will be missing. To provide such an overview and allow users to drill-down to each interested factor, we propose to construct a factor hierarchy based on the list of relevant keywords extracted. As shown in Figure 2, the factor hierarchy is composed of four levels: the top level are keyword clusters that include all extracted relevant keywords; the second level are keywords themselves; the third level are bigram phrases stemmed from relevant keywords; and the bottom level are individual documents (news, tweets, etc.) containing these

keywords/phrases. The lower two levels of the factor hierarchy can have overlaps in the same level. For example, one bigram phrase can yield two relevant keywords, and one document can have multiple bigram phrases. This factor hierarchy offers an initial overview of all factors relevant to stock price changes. The navigation on factor hierarchy through expand/collapse operations allows analyzing the details of every factor.

5 VISUALIZATIONS

Design Principle

The DeepClue interface is composed of four coordinated views, as shown in Figure 4(a)(b)(c)(d). We follow two principles in the visualization design. First, the visualization interface should help users complete three key tasks in the scenario of stock price prediction and analysis. Understanding stock market: The baseline task is to examine the underlying stock data, including price movements, trading volume, historical rise&fall trends, and the potential temporal patterns. Visualizing prediction result: Over the stock data, users should get access to the result produced by the model, i.e., whether a certain stock is predicted to rise or fall on the next day. S/he also needs to navigate the input data to the prediction model, i.e., the news/tweets collection in our scenario. Interpreting prediction model: Finally, users are expected to unveil the myth of the model by learning why and how each rise&fall prediction is decided. In DeepClue, this is achieved by visualizing the key textual factors that jointly make up the decision. Second, we design DeepClue for financial domain users, i.e., stock traders and investors. These users are mostly accustomed to classical financial visualization interfaces (e.g., Yahoo Finance), especially for the first two tasks in presenting stock data and their predictions. The classical visualization depends heavily on



Fig. 4: DeepClue interface: (a) stock timeline view showing the stock price history in an overview+detail design; (b) factor hierarchy view displaying the relevant textual factors to the stock prediction in a hierarchical structure; (c) document list view showing the related documents with the selected factor; (d) keyword map view depicting the relationship of relevant keywords.

statistical charts. Therefore, to reduce the user's learning cost, we build DeepClue from commodity statistical charts, both in the stock data and prediction visualization (Figure 4(a)(c)) and in illustrating their relevant predictive factors (Figure 4(b)(d)).

Visualization Components

In details, the stock timeline view in Figure 4(a) displays the price movement of a selected stock over time. This view is organized in an overview detail design to support flexible navigation of the timeline. The overview timeline chart on the top row allows users to specify a focused window on the timeline. The selected timeline is enlarged in the detail chart on the bottom row of Figure 4(a). On the detail timeline chart, four time series can be shown in two tabbed groups, as indicated by the draggable legends in the top left corner of the

detail timeline chart. The first tab on the left includes the actual stock price (solid line), the predicted stock price (dashed line), and the risk of the prediction (shading area). Note that there is little difference between these charts in Figure 4(a) because the selected timeline is almost ten years and the daily prediction introduces little variation. More separation of these timeseries can be observed in Figure 7 and Figure 10(a). In another legend tab on the right, the investment yield curve according to the prediction model is displayed, which is highly suggested in the expert study as the key indicator of model's success. As shown in Figure 9, the yield curve always starts from one. The model earns when the curve is above one and loses when it is below one.

User Interactions for Model Analysis

The basic user interactions in DeepClue are the customization of four coordinated views

according to the analysis task. On the stock timeline view, the “+” on the left panel allows users to juxtapose two bar chart time series for comparison purpose (Figure 4(a)). These bar charts can be dragged vertically to avoid overlaps with the timeline chart. In the factor hierarchy view, the hierarchical structure can be expanded and collapsed. Each factor can be deleted

6 EVALUATIONS

Learning from Financial News

In the first case, we study the use of DeepClue in interpreting the stock price prediction model over financial news. The details of the data set and model can be found in Section 3. We invited a stock trader from a private fund, one of our target users, to work with the DeepClue system. He was interested in investing on Apple Inc. (NASDAQ: AAPL), so the DeepClue configuration is set to display the prediction of Apple’s stock price from 2006 to 2015.

7. CONCLUSION

We present DeepClue, a system that visually interprets text based deep learning models in predicting stock price movements. DeepClue integrates three key designs from the cutting-edge deep learning technology: a hierarchical neural network model that embeds semantics in intermediate processing layers for interpretation; a backpropagation-like algorithm that effectively distributes the decision of prediction back to individual documents, bigrams and words; and an interactive visualization interface that allows users to navigate and analyze stock price timelines, textual factors, and their correlations. DeepClue has been deployed to predict S&P 500 stocks using mainstream financial news and firmspecific tweets. Both case studies, quantitative experiments, and the informal user study with domain experts demonstrate the usefulness of the proposed system in learning

from, evaluating and improving the text-based deep stock prediction models.

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